October 8, 2003
Name:

ATOC/ASTR 5560 Mid Term Test 1

Problem 1 (30 points).
The Microwave Water Radiometer measures brightness temperature of the sky at 23.8 GHz looking straight up from the ground. Calculate the integrated water vapor amount contained in tropical atmosphere, which corresponds to a measured brightness temperature of 64.3 K. The mean radiating temperature of the atmosphere is T_{mr}=288 K; the cosmic background at the top of atmosphere (which is a blackbody radiance at T_{cb}=2.7 K) is known, and the mass absorption coefficient is k_v=0.058 cm^2/g. Use the Rayleigh-Jeans approximation, i.e. use the brightness temperatures for all radiances.

Solution for problem 1.
The single layer thermal radiative transfer equation for down-welling radiance at μ=-1 using the Rayleigh-Jeans approximation can be written as following:

$$T_b = e^{-\tau_v} T_{cb} + (1 - e^{-\tau_v}) T_{mr}$$

The transmission factor can be derived as:

$$e^{-\tau_v} = \frac{T_{mr} - T_b}{T_{mr} - T_{cb}} .$$

Then, by taking a log function of both sides of equation we can derive optical depth τ_v:

$$\tau_v = -\log\left[\frac{T_{mr} - T_b}{T_{mr} - T_{cb}}\right].$$

The optical depth is product of absorption coefficient and optical path (or water vapor amount):

$$\tau_v = k_v u .$$

Therefore,

$$u = -\frac{1.}{0.058 cm^2 / g} \log\left[\frac{288K - 64.3K}{288K - 2.7K}\right] = 4.194 g / cm^2$$

Problem 2 (30 points).

Calculate the water vapor density for absorption lines at 183.3 GHz, 1560 cm^{-1}, and 1.37 μm, and find the approximate height from the standard atmosphere, where Lorentz and Dopler half-widths are the same. For each of these lines the Lorentz half-width at T_0=296 K and P_0=1.013x105 Pa is about α_0=0.10 cm-1. For water vapor R=461 J kg^{-1} K^{-1}. Assume, that n=0.5.

1976 Standard Atmosphere

Altitude [km]	**Temp** [Kelvin]	**Pressure** [pascal]	**H20 Density** [kg/m3]	**Air Density** [kg/m3]
0	288.15	101325	5.9E-3	1.225
5	255.65	54019.9121	6.4E-4	0.7361
10	223.15	26436.2676	1.8E-5	0.4127
15	216.65	12044.5709	7.2E-7	0.1937
20	216.65	5474.8887	4.0E-7	0.088
25	221.65	2511.0234	6.1E-7	0.0395
30	226.65	1171.8665	3.2E-7	0.018
35	237.05	558.9235	1.3E-7	0.0082
40	251.05	277.5216	4.8E-8	0.0039
45	265.05	143.1348	2.2E-8	0.0019
50	270.65	75.9448	7.8E-9	0.001
70	217.45	4.6342	1.2E-10	0.0001

Solution for problem 2.

The Doppler and Lorentz half-widths are defined as following:

$$\alpha_D = \nu\sqrt{\frac{2\ln 2 k_B T}{mc^2}} = \nu\sqrt{\frac{2\ln 2 R_{h2o} T}{c^2}}$$

$$R_{h2o} = R/m = RN_0/\mu$$

$$\alpha_L = \alpha_0 \frac{P}{P_0}\left(\frac{T_0}{T}\right)^n$$

To derive the air density at which the Lorentz and Doppler half-width are equal set $\alpha_L = \alpha_D$ and assume n=1/2. Use the gas law for air density:

$$\rho = \frac{P\mu_{air}}{RT} = \frac{\nu}{\alpha_0}\frac{P_0\mu_{air}}{RT_0}\sqrt{\frac{2\ln 2 R_{h2o} T_0}{c}}$$

The equal half-width density of air is

$$\rho = \frac{\nu}{\alpha_0}\left(1.19 kg/m^3\right)\left(1.45\times 10^{-6}\right)$$

The three following frequencies give the following wave-numbers and equivalent densities:

6.11 cm^{-1} $\rho_{air}=1.06\times10^{-4}$ kg/m^3 $\rho_{h20}=1.0\times10^{-7}$ g/m^3 z=65.0 km
1560 cm^{-1} $\rho_{air}=2.70\times10^{-2}$ kg/m^3 $\rho_{h20}=4.5\times10^{-4}$ g/m^3 z=27.5 km
7300 cm^{-1} $\rho_{air}=1.26\times10^{-1}$ kg/m^3 $\rho_{h20}=4.3\times10^{-4}$ g/m^3 z=18.5 km

Problem 3 (40 points).

In the table below are the K-distribution values for Fu and Liou's band 16 from 400 to 540 cm-1. The k-distribution values have been interpolated in pressure and temperature to the midpoints of the two layers in the mid-latitude winter atmosphere. The table also contains the absorber amount u of water vapor in each layer. Calculate the band mean zenith transmission from 0 to 4 km.

Layer (km)	U (kg/m^2)	K_1 (m^2/kg)	K_2	K_3	K_4	K_5	K_6	K_7
0-2	2.58	0.148	0.246	0.49	1.38	6.02	46.8	198
2-4	1.20	0.101	0.170	0.35	0.98	4.40	38.5	189
Δg_j		0.12	0.24	0.24	0.20	0.12	0.06	0.02

Solution to problem3.

Using k-distribution the band mean transmission is found by a weighted sum of transmissions (from τ_j) for each k:

$$T_{\Delta\nu} = \sum_{j=1}^{N} \Delta g_j \exp\left(-\sum_{l=1}^{L} k_j(z_l)\Delta u_l \right)$$

For the first k,

$$\tau_1 = k_1(0-2)u_{0-2} + k_1(2-4)u_{2-4} = 0.148(2.58) + 0.101(1.20) = 0.503$$

	1	2	3	4	5	6	7
τ_j	0.503	0.839	1.684	4.74	20.8	167	738
Exp($-\tau_j$)	0.65	0.432	0.186	0.009	0.0	0.0	0.0
Δg_j	0.12	0.24	0.24	0.20	0.12	0.06	0.02

The band transmission for the two layers is

$$\tau_{\Delta\nu} = \sum_{j=1}^{7} \Delta g_j \exp(-\tau_j) = 0.223$$

Problem 4 (additional 40 points).

Calculate the net flux divergences in the 400 to 500 cm^{-1} spectral band for the layer from 10 to 11 km in the mid-latitude summer atmosphere using the cooling to space approximation. Compare with net flux divergence obtained from the net fluxes in the table below. The table also contains flux transmissivities to space and the band integrated Plank functions.

Z (km)	P (mb)	T (K)	P (g/m^3)	$F_{net,\ 400\text{-}500}$ ($W\ m^{-2}$)	$T_{400\text{-}500}\ (z;\infty)$	$B_{400\text{-}500}\ (T)$ ($W\ m^{-2} sr^{-1}$)
10.0	281	235	417	28.13	0.9044	7.334
11.0	243	229	370	29.02	0.9535	6.793

Repeat calculations for the 980 to 1080 cm^{-1} spectral band, and compare to the previous case.

Z (km)	P (mb)	T (K)	P (g/m^3)	$F_{net,\ 980\text{-}1080}$ (Wm^{-2})	$T_{980\text{-}1080}\ (z;\infty)$	$B_{980\text{-}1080}\ (T)$ ($W\ m^{-2} sr^{-1}$)
10.0	281	235	417	21.51	0.5944	2.387
11.0	243	229	370	21.17	0.6022	1.024

Solution for problem 4

The cooling to space term of the heating rate equation is the product of the Plank flux emission, integrated over the spectral band, and the vertical gradient of the flux transmission:

$$\left.\frac{dF_{\Delta\nu,net}}{dz}\right|_{space} = \overline{B_{\Delta\nu}(z)} \times \pi \times \frac{\partial T_{\Delta\nu}(z;\infty)}{\partial z}$$

The derivative of the transmission is the weighting function referenced to space. This derivative may be done with a finite difference across the layer using the mean Plank function value to represent the layer average temperature:

$$\left.\frac{dF_{\Delta\nu,net}}{dz}\right|_{space} = (7.064 Wm^{-2} sr^{-1}) \times 3.14 \times \frac{0.9535 - 0.9044}{1000m} = 1.09 \times 10^{-3}\, Wm^{-3}$$

For comparison the net flux divergence from the net fluxes on each side of the layer is

$$\left.\frac{dF_{\Delta\nu,net}}{dz}\right|_{space} = \frac{F_{\Delta\nu,net}(z_2) - F_{\Delta\nu,net}(z_1)}{\Delta z} = 0.89 \times 10^{-3}\, Wm^{-3}$$

Thus the cooling to space approximation is fairly accurate for this layer and spectral region. It slightly overestimates the cooling, because the layer is heated as little by flux exchange with adjacent layers. The flux excheange with the surface is zero because there is no transmission to the surface due to strong water absorption in the pure rotation band of water vapor.

Using the same procedures as before, the flux divergence in this spectral band for the layer calculated using the cooling to space net approximation is:

$$\left.\frac{dF_{\Delta\nu,net}}{dz}\right|_{space} = (2.206 Wm^{-2} sr^{-1}) \times 3.14 \times \frac{0.6022 - 0.5944}{1000m} = 5.4 \times 10^{-5}\, Wm^{-3}$$

For comparison the net flux divergence from the net fluxes on each side of the layer is

$$\left.\frac{dF_{\Delta\nu,net}}{dz}\right|_{space} = \frac{F_{\Delta\nu,net}(z_2) - F_{\Delta\nu,net}(z_1)}{\Delta z} = -3.4 \times 10^{-4}\, Wm^{-3}$$

The cooling to space approximation fails in this case, since there is significant net flux convergence (heating). The spectral region is the 9.6 µm ozone band. There is little ozone in the lower troposphere, so the emission from the warm surface and lower tropospheric water vapor can reach the upper troposphere and lower stratosphere where it is absorbed, causing heating. The cooling to space is largely blocked by the ozone in the stratosphere above.